

MATHEMATICAL LOGIC — ASSIGNMENT THREE

- (1) Let α be an ordinal. Prove that $\alpha 1 = \alpha$.
- (2) State and prove Kleene's fixed point theorem.
- (3) Prove that the Axiom of Choice on a finite family is derivable in ZF: every finite collection $\{X_i\}_{1 \leq i \leq n \in \mathbb{N}}$ of non-empty sets has a choice function $f: \{X_i\}_{1 \leq i \leq n} \rightarrow \bigcup_{1 \leq i \leq n} X_i$ such that $f(X_i) \in X_i$ for every $1 \leq i \leq n$.

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.

Date: May 23th, 2022.