

## MATHEMATICAL LOGIC — ASSIGNMENT THREE

- (1) Let  $A$  be an infinite set: show that the cardinality of the set of functions from  $A$  to  $2$  is the same of  $\wp(A)$ .

Consider the function  $f$  mapping  $g: A \rightarrow 2$  to  $\{x: g(x) = 1\}$ .

If  $f(g) = f(h)$  then  $\{x: g(x) = 1\} = \{x: h(x) = 1\}$ , thus  $\{x: g(x) = 0\} = \{x: h(x) = 0\}$ , so  $g(x) = h(x)$  for every  $x \in A$ , hence  $g = h$ , that is  $f$  is injective.

If  $B \in \wp(A)$  then  $B \subseteq A$ , so the function  $A \rightarrow 2$  defined by  $x \in B \mapsto 1$  and  $x \in A \setminus B \mapsto 0$  is mapped into  $B$  by  $f$ , that is,  $f$  is surjective.

- (2) State and prove Cantor's theorem.

See Theorem 15.24 in the slides.

- (3) Show a non-strongly normalising formal type system.

Consider the type system with just one type:  $\Lambda$  and whose terms are all the  $\lambda$ -terms.

Then,  $\Omega: \Lambda$  but  $\Omega$  reduces to itself in one step, so every reduction sequence from it can be indefinitely extended.

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.

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