# MATHEMATICAL LOGIC — ASSIGNMENT ONE

(1) Prove that $A \supset B$ is equivalent to $\neg B \supset \neg A$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[A \supset B]^1 \quad [A]^2}{B}{\scriptstyle \supset\mathrm{E}} \qquad [\neg B]^3}{\bot}{\scriptstyle \neg\mathrm{E}}}{\neg A}{\scriptstyle \neg\mathrm{I}^2}}{\neg B \supset \neg A}{\scriptstyle \supset\mathrm{I}^3}}{(A \supset B) \supset (\neg B \supset \neg A)}{\scriptstyle \supset\mathrm{I}^1}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{B \vee \neg B}{\scriptstyle \mathrm{lem}} \qquad [B]^1 \qquad \dfrac{\dfrac{\dfrac{[\neg B \supset \neg A]^2 \quad [\neg B]^1}{\neg A}{\scriptstyle \supset\mathrm{E}} \qquad [A]^3}{\bot}{\scriptstyle \neg\mathrm{E}}}{B}{\scriptstyle \bot\mathrm{E}}}{B}{\scriptstyle \vee\mathrm{E}^1}}{A \supset B}{\scriptstyle \supset\mathrm{I}^3}}{(\neg B \supset \neg A) \supset (A \supset B)}{\scriptstyle \supset\mathrm{I}^2}$$

(2) Show that in any bounded distributive complemented lattice, each element has a unique complement.

This is Proposition 5.13 in the slides.

(3) Consider the calculus in which the $\neg$I inference rule is absent. Is it sound and complete with respect to the Boolean algebra semantics?

The $\neg$I inference rule:

$$\dfrac{\begin{matrix}[A]^1 \\ \vdots \\ \bot\end{matrix}}{\neg A}{\scriptstyle \neg\mathrm{I}^1}$$

if absent, can be simulated by

$$\dfrac{\dfrac{}{A \vee \neg A}{\scriptstyle \mathrm{lem}} \qquad \dfrac{\begin{matrix}[A]^1 \\ \vdots \\ \bot\end{matrix}}{\neg A}{\scriptstyle \bot\mathrm{E}} \qquad [\neg A]^1}{\neg A}{\scriptstyle \vee\mathrm{E}^1}$$

Hence, the usual natural deduction system and this reduced system, prove the same set of theorems, and thus the latter is sound and complete because the former is so.

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.

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