MATHEMATICAL LOGIC — ASSIGNMENT THREE

(1) Prove that every infinite ordinal can be written as the ordinal sum of a limit ordinal and a finite ordinal.

By transfinite induction from ω:
- If α is limit then $\alpha = \alpha + 0$;
- if $\alpha = \beta + 1$, then, by induction hypothesis, $\beta = \delta + n$ with δ a limit ordinal, and n a finite ordinal; thus, $\alpha = \delta + (n + 1)$.

(2) State and prove the Schröder-Bernstein Theorem-

See Theorem 11.1 in the slides.

(3) Illustrate the Gödel-Gentzen translation. Show two formulae which are equivalent in classical logic but not equivalent in intuitionistic logic.

The first part of the exercise is Definition 15.2 in the slides. As for the second part, it suffices to take $A \vee \neg A$ and $(A \vee \neg A)^N$ in the propositional fragment: by Proposition 15.4, the latter is provable in intuitionistic logic, while the former is not even valid because of the Completeness Theorem and Fact 16.4. Hence, they are not intuitionistically equivalent. However, by Proposition 15.3, these two formulae are equivalent in classical logic.

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.

Date: January 10^{th}, 2019.