MATHEMATICAL LOGIC — ASSIGNMENT ONE

(1) Prove that $(A \supset B) = (\neg A \lor B)$.

$$\dfrac{\dfrac{\dfrac{}{A \lor \neg A}\,\text{lem} \qquad \dfrac{\dfrac{[A \supset B]^1 \quad [A]^2}{B}\,\supset\text{E}}{\neg A \lor B}\,\lor\text{I}_2 \qquad \dfrac{[\neg A]^2}{\neg A \lor B}\,\lor\text{I}_1}{\neg A \lor B}\,\lor\text{E}^2}{(A \supset B) \supset \neg A \lor B}\,\supset\text{I}^1$$

$$\dfrac{\dfrac{\dfrac{[\neg A \lor B]^1 \qquad \dfrac{\dfrac{[\neg A]^2 \quad [A]^3}{\bot}\,\neg\text{E}}{B}\,\bot\text{E} \qquad [B]^2}{B}\,\lor\text{E}^2}{A \supset B}\,\supset\text{I}^3}{\neg A \lor B \supset (A \supset B)}\,\supset\text{I}^1$$

(2) Show that, if a lattice is distributive then both the laws

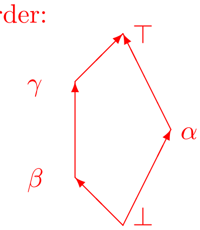
$$x \land (y \lor z) = (x \land y) \lor (x \land z)$$
$$x \lor (y \land z) = (x \lor y) \land (x \lor z)$$

hold.

The first law is valid by definition. For the second one, see Proposition 5.12 in the slides.

(3) Show an example of non-distributive lattice. Prove that the distributive law fails on it.

Consider the following order:



By direct inspection, this is a bounded lattice, since every pair of elements has a meet and a join, and $\top$ and $\bot$ are the maximum and the minimum or the order, respectively.

However,

$$\gamma \land (\beta \lor \alpha) = \gamma \land \top = \gamma$$
$$(\gamma \land \beta) \lor (\gamma \land \alpha) = \beta \lor \bot = \beta \ .$$

Date: October 31^{tst}, 2018.

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.