Mathematical Logic — Assignment Four

June 8^{th}, 2018

1. Illustrate an alternative coding of partial recursive functions.

   The coding is based on the unique factorisation in primes of every natural numbers. By permuting the prime factors used in the coding seen during the course, we get an alternative coding.

2. State one *natural* incompleteness result.

   See. e.g., Theorem 21.4, slide 473 or what follows.

3. Consider Gödel's Incompleteness Theory: *A first-order theory which (i) is effective, (ii) represents all the partial recursive functions, (iii) is consistent, is necessarily incomplete.* Can you drop any of the assumptions and still get the conclusion? Motivate your answer.

   The collection of all the true formulae on the natural numbers with the same signature as Peano arithmetic is able to represent all the partial recursive functions, and it is consistent. Evidently, it is not effective. However, it is also complete! So, condition (i) cannot be dropped.

   The empty theory is effective and consistent, and it corresponds to the pure logic, which is complete. Evidently, it does not allow to represent the partial recursive functions. Hence condition (ii) cannot be dropped.

   The theory containing all the formulae which could be written in the language of Peano arithmetic is clearly effective and represents all the recursive functions. Clearly, it is not consistent, and, obviously, it is complete in a trivial sense, so condition (iii) cannot be dropped.

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.