Mathematical Logic — Assignment Three

May 25th, 2018

1. Prove that $SKK = I$ in the λ-calculus.

   $SKK = (\lambda x, y, z.\, (xz)(yz))KK = \lambda z.\, (Kz)(Kz) = \lambda z.\, ((\lambda x, y.\, x)z)(Kz) = \lambda z.\, (\lambda y.\, z)(Kz) = \lambda z.\, z = I.$

2. Illustrate the so-called Curry-Howard isomorphism.

   See slides 402 and 403.

3. Show that intuitionistic propositional logic plus $\neg\neg A = A$ yields $\neg\neg(A \lor \neg A)$.

   (Hint: prove that $\neg(x \lor y) = \neg x \land \neg y$ holds in intuitionistic logic.)

   Following the hint:

$$\dfrac{\dfrac{\dfrac{[x \lor y]^1 \quad \dfrac{[x]^2 \quad \dfrac{[\neg x \land \neg y]^3}{\neg x}{\scriptstyle \land\mathrm{E}_1}}{\bot}{\scriptstyle \neg\mathrm{E}} \quad \dfrac{[y]^2 \quad \dfrac{[\neg x \land \neg y]^3}{\neg y}{\scriptstyle \land\mathrm{E}_2}}{\bot}{\scriptstyle \neg\mathrm{E}}}{\bot}{\scriptstyle \lor\mathrm{E}^2}}{\neg(x \lor y)}{\scriptstyle \neg\mathrm{I}^1}}{\neg x \land \neg y \supset \neg(x \lor y)}{\scriptstyle \supset\mathrm{I}^3}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{[\neg(x \lor y)]^1 \quad \dfrac{[x]^2}{x \lor y}{\scriptstyle \lor\mathrm{I}_1}}{\bot}{\scriptstyle \neg\mathrm{E}}}{\neg x}{\scriptstyle \neg\mathrm{I}^2} \quad \dfrac{\dfrac{[\neg(x \lor y)]^1 \quad \dfrac{[y]^3}{x \lor y}{\scriptstyle \lor\mathrm{I}_2}}{\bot}{\scriptstyle \neg\mathrm{E}}}{\neg y}{\scriptstyle \neg\mathrm{I}^3}}{\neg x \land \neg y}{\scriptstyle \land\mathrm{I}}}{\neg(x \lor y) \supset \neg x \land \neg y}{\scriptstyle \supset\mathrm{I}^1}$$

   Hence $\neg\neg(A \lor \neg A) = \neg(\neg A \land \neg\neg A) = \neg(\neg A \land A) = \neg\bot = \top$.

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.