Mathematical Logic — Assignment Two

December 5^{th}, 2016

1. Prove that $\vdash \neg\neg(\forall x.\, A) \supset (\forall x.\, \neg\neg A)$.
2. Show that, for any set X, there is a bijective correspondence between X and $|X|$, using the Axiom of Choice.
3. Consider the signature with addition, multiplication, 0, 1, and the $<$ relation, and let $\mathbb{R}$ be the real numbers with the standard interpretation of the symbols. Let T be the set of all true sentences (formulas with no free variables) in the structure $\mathbb{R}$ on that signature. Show that T has a model in which there are infinitesimal numbers, i.e. there is c for which $0 < c < 1/n$ for every $n \in \mathbb{N}$.

   (This exercise will be evaluated considering, in the first place, how you reason, so be more focused in giving a plausible answer, rather than trying hard to find the correct one)

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.