Mathematical Logic — Assignment Four

February 1st, 2017

1. Show that there is a non-standard model of Peano arithmetic.

   See Proposition 19.3, slide 433.

2. Write the statement of one *natural* incompleteness result.

   See. e.g., Theorem 21.4, slide 473 or what follows.

3. Discuss whether a theory of real numbers allows to prove any true first-order sentence on $\mathbb{R}$, the set of real numbers.

   Let T be a first-order theory having $\mathbb{R}$ as a model. If T is both effective and able to represent all the recursive functions then, by Gödel's incompleteness theorem, there is a formula which is both true and non provable. Otherwise, when T is effective but does not represent all the recursive functions, there is a sentence, representing some recursive function, which cannot be derived in T.

   If we drop the requirement to be effective, the theory which comprehends all the true formulae on $\mathbb{R}$, is obviously able to prove all of them, trivially.

Each question is worth 12 points. The points in all the four assignments will be added together and the result will be divided by 4, and this will be the final result. Remember to mark your answer sheet with your name.