Strong Constructivity of Second-Order Intuitionistic Arithmetic

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The goal of this talk is to show that second-order intuitionistic arithmetic is strongly constructive. Although this result is by no means surprising, the proving technique we use is new, and far more general than the result we get.

We say that a logical theory Γ is naively constructive when:

- if $\Gamma \vdash A \vee B$ then $\Gamma \vdash A$ or $\Gamma \vdash B$ where $A \vee B$ is a closed formula (disjunction property).
- if $\Gamma \vdash \exists x.A(x)$ then there is a closed term t such that $\Gamma \vdash A(t)$, where $\exists x.A(x)$ is a closed formula (explicit definability property).

A theory is said to be strongly constructive [Fer97,FM97], if it is naively constructive and any proof of $A \vee B$ ($\exists x.A(x)$, respectively) contains enough information to build up a proof of A or a proof of B (or a proof of $A(t)$, for a suitable closed term t, respectively).

We will use the so called Collection Method [MO81,MO79,Ben97] to show that second-order intuitionistic arithmetic is strongly constructive.

The Collection Method is a proof theoretical instrument especially built to characterize the notion of strong constructivity. It has been used to give a computational meaning to constructive proofs, and to prove that many logics are constructive. Since this instrument is relatively new and it succeeds to prove constructivity for a logic (theory) even when semantical methods are not applicable (lacking a semantical characterization of models, for example), or when the logical system is not cut-free, we think it is worthwhile showing an application even if the result is well known.

References

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